TRANSPORT AND ASYMMETRIES IN SPATIAL COMPETITION (1)

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1 — Introduction

The spatial competition litterature provides a fairly good description of the interaction of two selllers located in a market segment and competing for the consumers distributed along that segment. The game is usually modeled as a two-stage game. In the first stage the firms select locations anticipating the competition in prices, as in Hotelling (1929), or in quantities, as described in Beckmann and Thisse (1986 pp. 47-49), in the second. This litterature turns mainly around two questions: does the locational pattern of the firms in equilibrium entail minimum or maximal differentiation? In the case of price competition, is there a Nash price equilibrium for all firms' locations, i. e. is there stability in competition? Both questions were addressed by Hotelling in his 1929's paper and the latter question engendered an important research field which began with D'Aspremont, Gabszewicz and Thisse (1979).

This paper tries to deal with some aspects of spatial competition which have been less discussed by the existing litterature.

Spatial competition models are usually static. The impact of the improvement of transport technologies which appears through decaying transport rates in time is seldom dealt, although the subject has been introduced by Launhardt's (1885) seminal work and it is implicitly present in Smithies (1941).

Most spatial oligopoly models are symmetric, both in what concerns the productive efficiency of firms and the spatial distribution of consumers.

Lastly, most spatial competition models use only one kind of competition in the second stage of the game, either price or quantity competition.

Therefore the paper seeks to describe the impact of the decay of the transport rate, due to the improvement of transport, on the equilibrium of a spatial oligopoly in the context of a comparative static framework. This impact is seen to depend critically on the existence of asymmetry either in the unit production costs or in the spatial distribution of consumers. This result is robust with relation to the kind of competition retained (price or quantity competition).

The first part of the paper contains a spatial duopoly inspired by Launhardt (1885) where the price-competing firms locate at the extreme points of the unit interval, along which consumers are distributed with uniform density. When the transport rate decreases, customers are transferred from the relatively inefficient firm to the more efficient one. If following Dos Santos Ferreira and Thisse

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(1992), we assume that the firms play a two-stage game where the firms choose first specific transport rates in a bounded interval and then compete in price, the choice of the lower bound transport rate by the more efficient firm depends on the existence of a sufficient level of asymmetry between consumers.

The second part of the paper, which is remotely inspired in Smithies (1941), features competition in quantity by two firms which locate in a set of two towns with different consumer populations connected by a transport line. Consumers have price elastic demand functions. If the distribution of consumers in space is very asymmetric, both firms locate in the larger town for every value of the transport rate. In the case where the spatial distribution of consumers is more balanced, two alternative outcomes may arise. If the transport cost is high, the firms locate in different towns, with each firm oriented towards local demand. When the transport cost decreases below a certain critical level (which is seen to depend inversely on the ratio of the population of the smaller town to the population of the larger town), both firms locate in the larger town, thus being oriented towards the central point of the spatial distribution of consumers.

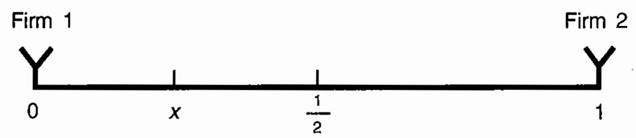
2 — Transport and asymmetry in productive efficiency

We suppose a market situation which was described by Launhardt (1885). The assumptions are:

1) The spatial market is the interval [0,1];
2) Consumers are uniformly distributed along the market with unit density;
3) Two firms producing a homogeneous product locate at the extreme points of the interval;
4) Firms compete through the quotation of fob mill prices p_1 and p_2;
5) Each consumer pays a delivered price which is equal to the sum of mill price and transport cost between firm and consumer locations;
6) Transport cost between the firm and the consumer corresponds to the product of a constant transport rate t by the distance between the firm and the consumer;
7) Each consumer buys a unit of product per unit of time;
8) Each consumer buys the product with the lowest delivered price;
9) Firms have different unit production costs c_1 and c_2, with $c_1 > c_2$ without loss of generality.

FIGURE 2.1

Spatial market



Consumers are divided into market areas whose border is point x, characterized by the equality of delivered prices:

$$p_1 + tx = p_2 + t(1 - x) \qquad \textbf{2.1}$$

Once solved **2.1** becomes:

$$x = \frac{P_2 - P_1}{2t} + \frac{1}{2} \qquad \textbf{2.2}$$

We remark that $x = D_1(p_1,p_2)$ (demand addressed to firm 1) and $1 - x = D_2(p_1,p_2)$ (demand addressed to firm 2). Profit functions are:

$$\pi_1 = (p_1 - c_1)D_1(p_1,p_2) \qquad \textbf{2.3}$$

$$\pi_1 = (p_2 - c_2)D_2(p_1,p_2) \qquad \textbf{2.4}$$

We can find Nash equilibrium prices as functions of t:

$$p^*_1 = t + \frac{2c_1 + c_2}{3} \qquad \textbf{2.5}$$

$$p^*_2 = t + \frac{2c_2 + c_1}{3} \qquad \textbf{2.6}$$

It is obvious that p^*_1 and p^*_2 are strictly increasing functions of t: increasing transport cost decreases the degree of substitution between products and raises prices. Diminishing transport cost increases the intensity of competition with a negative effect on prices.

Substituting p^*_1 and p^*_2 from **2.5** and **2.6** in **2.2**, we obtain the border of the market areas as a function of t.

$$x(t) = \frac{c_2 - c_1}{6t} + \frac{1}{2} \qquad \textbf{2.7}$$

This function is depicted below in figure 2.2.

Function $x(t)$ is increasing, concave and converges asymptotically to $\frac{1}{2}$. The function reaches a zero at:

$$t = \frac{c_2 - c_1}{6t} \qquad \textbf{2.8}$$

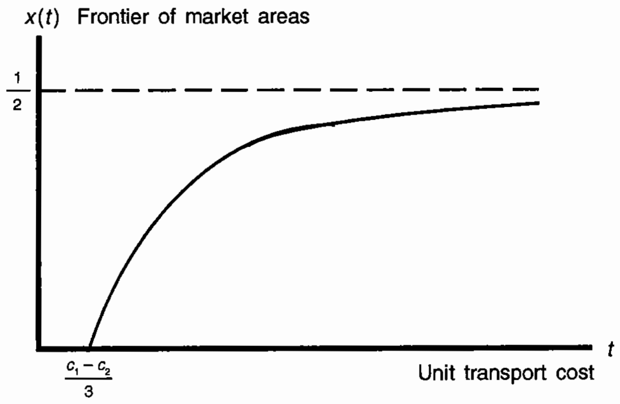
As was remarked by Launhardt (1885), the improvement in transport shifts customers from the relatively inefficient to the low cost firm.

How does each firm's profit change with the improvement in transport?

1) The improvement in transport unambiguously decreases the profit of the less efficient firm because it decreases both the firm's price and its market share.

FIGURE 2.2

***x* (*t*) function**



2) The effect of the improvement in transport on the profit of the more efficient firm is ambiguous because it decreases the firm's price with a negative impact on profit and it increases its market area with an opposite effect.

Substituting the Nash equilibrium prices from **2.5** and **2.6** in the more efficient firm's profit function (expressed by **2.4**) we obtain a function $\pi_2(t)$:

$$\pi_2(t) = \left(\frac{1}{2} + \frac{c_1 + c_2}{6t}\right)\left(t + \frac{2c_2 + c_1}{3} - c_2\right) \qquad \textbf{2.9}$$

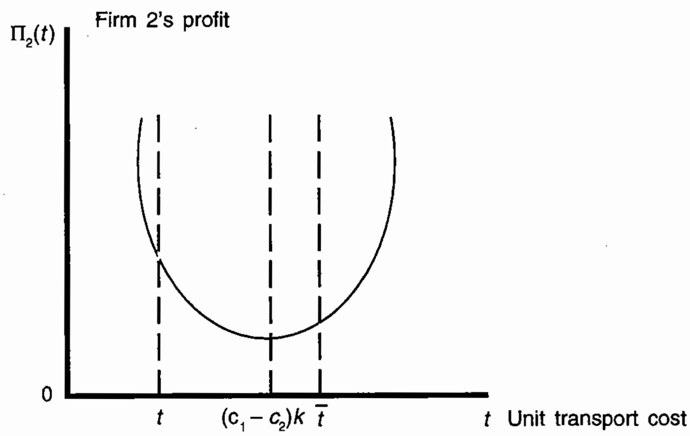
Function $\pi_2(t)$ is represented in figure 2.3. It is convex and has a minimum at:

$$t = (c_1 - c_2)k \qquad \textbf{2.10}$$

where $k = \frac{1}{3}$.

FIGURE 2.3

Function $\pi_2(t)$



It can be noticed that when the transport cost is high in relation to the difference between the unit production costs, the effect of the decrease in transport cost on firm 2' s profit is negative: the price effect overcomes the positive effect on market share. On the other hand, when the difference between the unit production costs is high in relation to transport cost, the latter's decrease enhances firm 2' s profit: the positive effect on the market share overcomes the negative effect on price.

Following Dos Santos Ferreira and Thisse (1992), we can endogenise the selection of transport technology by assuming that the firms play a two-stage game:

First stage: firms select simultaneously their specific transport technologies, as expressed by transport rates t_1 and t_2. Transport rates are bounded to interval $[\underline{t},\overline{t}]$;

Second stage: firms quote simultaneously prices p_1 and p_2.

We study the equilibrium of this game in appendix A. We conclude there that to select the upper bound transport rate t is a dominant strategy to the less efficient firm in the subgame of transport rates. Therefore the solution of the game can studied by means of the payoff function of the efficient firm only. In what follows, we present a heuristic version of the game, by supposing that there is a unique transport rate in the market which is selected by the more efficient firm. The more efficient firm chooses his transport technology and the less efficient firm is constrained to follow it. The heuristic game can be modeled in the following way:

First stage: the most efficient firm selects a transport technology expressed by t. The choice of t is bounded to the interval $[t,\overline{t}]$;

Second stage: the firms compete in prices.

Subgame perfect equilibrium is the pertinent concept of equilibrium.

As the function depicted in figure 2.3 is convex and is defined in a closed interval, it reaches the maximum at an extreme point of the interval. A sufficient (although not necessary) condition for it to reach a maximum at t is:

$$\overline{t} \leq (c_1 - c_2)k$$

or: **2.11**

$$c_1 - c_2 \geq \frac{\overline{t}}{k}$$

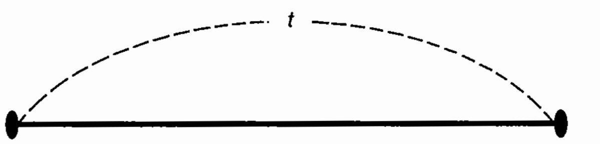
The above expression means that the most efficient transport technology will be chosen in equilibrium if the difference of productive efficiency is high enough. In this case, market areas in equilibrium will be very unequal, the most efficient firm supplying almost the whole market. On the contrary, if production cost differences are relatively low, the high transport cost technology is chosen and market areas become alike, the most efficient firm supplying slightly more customers than the other firm.

3 — Transport and asymmetry in the spatial distribution of demand

Represents the spatial structure of the economy:

FIGURE 3.1

Spatial structure of the economy



The assumptions are:

1) There exist two towns that are connected by a transport line;
2) All consumers and firms locate in the towns;
3) Two firms produce and distribute a homogeneous good;
4) The transport cost of the good in the distance between the two towns is named t;
5) The population of town A, n_a, is higher than the population of town B, n_b;
6) Each consumer's inverse demand function is $p = 1 - q$;
7) Firms' unit production costs are equal (equal to 0, to simplify).

We consider a two-stage game:

First stage: firms choose locations;
Second stage: firms compete through the selection of quantities sold in each town.

The payoff matrix in the first stage is:

Firm 2		Town A	Town B
Firm 2			
	Town A	$\pi_1(A,A)$; $\pi_2(A,A)$	$\pi_1(A,B)$; $\pi_2(A,B)$
	Town B	$\pi_1(B,A)$; $\pi_2(B,A)$	$\pi_1(B,B)$; $\pi_2(B,B)$

The following relations stem from the symmetry of the firms:

$\pi_1(B,A) = \pi_2(A,B)$;
$\pi_2(B,A) = \pi_1(A,B)$;
$\pi_1(A,A) = \pi_2(A,A)$;
$\pi_1(B,B) = \pi_2(B,B)$.

Because of the symmetry of the firms, we only need to study the quantity and profit equilibria in two cases: the two firms have identical locations (e. g., they locate in A; but if they locate in B the same results would hold); the firms have different locations (e. g., firm 1 locates in A and firm 2 in B).

Terminology:

q_{1a}, q_{1b}, q_{2a},q_{2b} — quantity sold by a firm (1,2) in a market (a,b);
π_{1a}, π_{1b}, π_{2a}, π_{2b} — profits generated by sales q_{1a}, q_{1b}, q_{2a},q_{2b};
$\pi_1 = \pi_{1a} + \pi_{1b}$; $\pi_2 = \pi_{2a} + \pi_{2b}$ — aggregate profits of firm 1 and 2;
$Q_a = q_{1a} + q_{2a}$; $Q_b = q_{1b} + q_{2b}$ — aggregate quantities sold in towns A and B.

First we consider the case corresponding to locations (A,A). In market A, profit functions of the firms are:

$$\pi_{1a}(q_{1a},q_{2a}) = q_{1a}\left(1 - \frac{q_{1a} + q_{2a}}{n_a}\right) \qquad \textbf{3.1}$$

$$\pi_{2a}(q_{1a},q_{2a}) = q_{2a}\left(1 - \frac{q_{1a} + q_{2a}}{n_a}\right) \qquad \textbf{3.2}$$

A Cournot-Nash equilibrium follows:

$$q^*_{1a} = q^*_{2a} = \frac{n_a}{3} \qquad \textbf{3.3}$$

Profits which correspond to the equilibrium quantities in **3.3** are:

$$\pi^*_{1a} = \pi^*_{2a} = \frac{n_a}{9} \qquad \textbf{3.4}$$

In market B profit functions are:

$$\pi_{1b}(q_{1b},q_{2b}) = q_{1b}\left[\left(1 - \frac{q_{1b} + q_{2b}}{n_b}\right) - t\right] \qquad \textbf{3.5}$$

$$\pi_{2b}(q_{1b},q_{2b}) = q_{2b}\left[\left(1 - \frac{q_{1b} + q_{2b}}{n_b}\right) - t\right] \qquad \textbf{3.6}$$

The Cournot-Nash equilibrium is:

$$q^*_{1b} = q^*_{2b} = \frac{n_b(1-t)}{3} \qquad \textbf{3.7}$$

The profit functions in equilibrium are:

$$\pi^*_{1b} = \pi^*_{2b} = \frac{n_b(1-t)^2}{9} \qquad \textbf{3.8}$$

We infer from **3.3** and **3.7** and from **3.4** and **3.8** that total quantities sold by each firm and their total profits are monotonic decreasing functions of the transport cost. We remark that the condition $t < 1$ should hold for sales and profits in the distant market (B) to be positive.

Now we consider now the case (A,B): firm 1 locates in town A and firm 2 is based in town B.

Profit functions in town A are:

$$\pi_{1a}(q_{1a},q_{2a}) = q_{1a}\left(1 - \frac{q_{1a}+q_{2a}}{n_a}\right) \tag{3.9}$$

$$\pi_{2a}(q_{1a},q_{2a}) = q_{1a}\left[\left(1 - \frac{q_{1a}+q_{2a}}{n_a}\right) - t\right] \tag{3.10}$$

Profit functions in town B are:

$$\pi_{1b}(q_{1b},q_{2b}) = q_{1b}\left[\left(1 - \frac{q_{1b}+q_{2b}}{n_b}\right) - t\right] \tag{3.11}$$

$$\pi_{1b}(q_{1b},q_{2b}) = q_{1b}\left(1 - \frac{q_{1b}+q_{2b}}{n_b}\right) \tag{3.12}$$

The expressions of Nash equilibrium quantities depend on whether transport cost is higher or lower than $\frac{1}{2}$. For $t > \frac{1}{2}$, each firm sells only in its local market. For $t < \frac{1}{2}$, each firm exports to the distant market thus generating positive cross flows. Therefore, for q^*_{1a} we have:

$$q^*_{1a} = \frac{n_a(1+t)}{3} \text{ for } t \leq \frac{1}{2} \tag{3.13}$$

$$q^*_{1a} = \frac{n_a}{2} \text{ for } t > \frac{1}{2} \tag{3.14}$$

For q^*_{2a} we have:

$$q^*_{2a} = \frac{n_a(1-2t)}{3} \text{ for } t \leq \frac{1}{2} \tag{3.15}$$

$$q^*_{2a} = 0 \text{ for } t > \frac{1}{2} \tag{3.16}$$

Functions $q^*_{1a}(t)$ and $q^*_{2a}(t)$ from **3.13** to **3.16** are both continuous, the former increasing and the latter decreasing in t. The improvement in transport determines the substitution of local products by imported products from the other town.

By symmetry, through exchanging subscripts a and b, and also 1 and 2 in **3.13** to **3.16**, we also obtain the equilibrium quantities in market B. q^*_{1b} is:

$$q^*_{1b} = \frac{n_b(1-2t)}{3} \text{ for } t \leq \frac{1}{2} \tag{3.17}$$

$$q^*_{1b} = 0 \text{ for } t > \frac{1}{2} \tag{3.18}$$

While q^*_{2a} is:

$$q^*_{2b} = \frac{n_b(1+t)}{3} \text{ for } t \leq \frac{1}{2} \qquad \textbf{3.19}$$

$$q^*_{2b} = \frac{n_b}{2} \text{ for } t > \frac{1}{2} \qquad \textbf{3.20}$$

The aggregate quantity consumed in equilibrium in market *A* is by definition:

$$Q^*_a \equiv q^*_{1a} + q^*_{2a}$$

From **3.13** to **3.16**, we have:

$$Q^*_a = \frac{n_a}{2} \text{ for } t > \frac{1}{2} \qquad \textbf{3.21}$$

$$Q^*_a = \frac{n_a(2-t)}{3} \text{ for } t \leq \frac{1}{2} \qquad \textbf{3.22}$$

Function $Q^*_a(t)$ [and the same for $Q^*_b(t)$] is continuous and decreasing: consumption grows in each market as a result of the transport improvement.

Firm 1' s equilibrium profit which is raised in the two markets is by definition:

$$\pi^*_1 \equiv \pi^*_{1a} + \pi^*_{1b}$$

π^*_{1a}, firm 1' s profit from sales in market *A* is easily calculated by substituting **3.13** to **3.16** in **3.9**, with the result that:

$$\pi^*_a = \frac{n_a}{4} \text{ for } t > \frac{1}{2} \qquad \textbf{3.23}$$

$$\pi^*_a = \frac{n_a(1+t)^2}{9} \text{ for } t \leq \frac{1}{2} \qquad \textbf{3.24}$$

On the other hand, π^*_b can be found substituting **3.17** to **3.20** in **3.11**, with it thus resulting that:

$$\pi^*_{1b} = 0 \text{ for } t > \frac{1}{2} \qquad \textbf{3.25}$$

$$\pi^*_{1b} = \frac{n_b(1-t)^2}{9} \text{ for } t \leq \frac{1}{2} \qquad \textbf{3.26}$$

Through aggregation of **3.23** and **3.25** and of **3.24** and **3.26** firm 1' s total profit achieved in both towns is:

$$\pi^*_1 = \frac{n_a}{4} \text{ for } t > \frac{1}{2} \qquad \textbf{3.27}$$

$$\pi^*_1 = \frac{n_a(1+t)^2 + n_b(1-t)^2}{9} \text{ for } t \leq \frac{1}{2} \qquad \textbf{3.28}$$

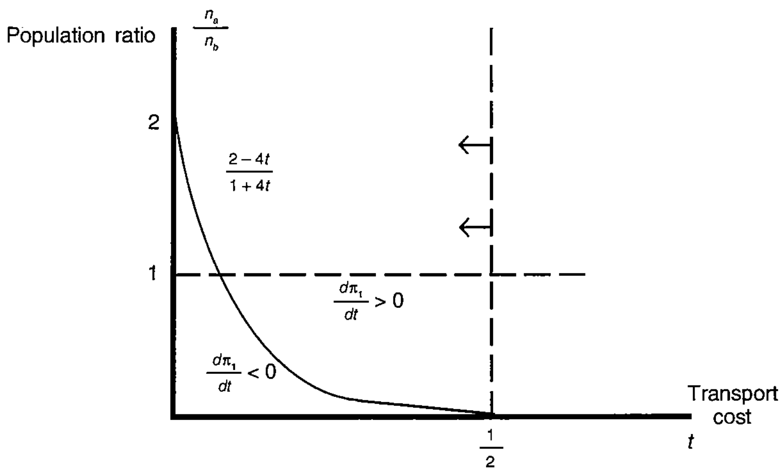
It is easy to show that for $t < \frac{1}{2}$:

$$\frac{d\pi^*_1}{dt} > 0 \text{ if } \frac{n_a}{n_b} > \frac{2-4t}{1+t} \qquad \textbf{3.29}$$

The right hand side of inequality **3.29** is an increasing and convex function whose zero occurs in $t = \frac{1}{2}$ and that takes value 2 for $t = 0$ (cf. figure **3.2**).

FIGURE 3.2

The direction of change in firm 1' s profit with a decrease in transport cost



We infer from figure 3.2 that firm 1' s profit (firm 2 being located in B) decreases monotonically with the transport cost if $\frac{n_a}{n_b} > 2$. If $1 < \frac{n_a}{n_b} < 2$ holds, firm 1' s profit first decreases and then increases when t falls below $\frac{1}{2}$. With the application of a symmetric reasoning, the latter assertion holds for firm 2' s profit. This behaviour arises from two conflicting effects:

The improvement of transport expands quantities sold in each market, which has a positive effect on profit

The transport improvement raises freight through the substitution of outside market sales for sales in the local market with a negative effect on profit.

Two propositions can be demonstrated concerning the equilibrium of locations in the first stage game.

Proposition 1 — (B,B) is not a location equilibrium (v. proof in appendix B)

This proposition means that when both firms locate in the smaller market it always pays off for each of them to move unilaterally to the larger market.

Proposition 2 — If the distribution of the population between the markets is very asymmetric, e. g. if $\frac{n_b}{n_a} \leq \frac{4}{9}$, the equilibrium of locations is (A,A) for every value of the transport cost. If $\frac{n_b}{n_a}$ belongs to the interval $\left[\frac{4}{9}, \frac{1}{2}\right]$, the equilibrium of locations will be (A,B) if:

$$t \geq 1 - \sqrt{\frac{9}{4} - \frac{n_a}{n_b}}$$

and it will be (A,A) otherwise. If $\frac{n_b}{n_a}$ belongs to the interval $\left[\frac{1}{2}, 1\right]$ the equilibrium of locations will be (A,B) if:

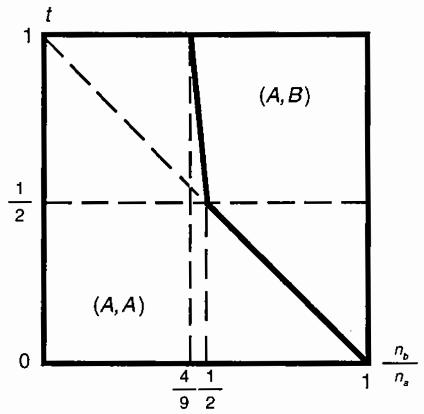
$$t \geq 1 - \frac{n_b}{n_a} \text{ (v. proof in appendix C)}$$

and will be (A,A) otherwise.

The parameter space is depicted in figure 3.3:

FIGURE 3.3

The space of parameters



The proposition means that when the distribution of population is very asymmetric, both firms locate in equilibrium in the larger market. If the distribution of population is more balanced, the equilibrium of locations depends on the transport cost. When the transport cost is high, the equilibrium of locations entails a firm in each market, so that the firms are oriented towards local demand. For low values of transport cost, production is concentrated in the larger town, so that production is oriented to the central point of the spatial distribution of consumers in order to maximize accessibility to demand in the whole market (2).

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(2) More precisely, firms locate at the median and mode of the distribution of consumers rather than at the mean, which is an intermediate point between A and B that is excluded by assumption.

4 — Conclusions

In a spatial oligopoly where there exist differences of unit production costs among the firms, a decrease in transport cost shifts customers away from the less efficient firm to the lower cost one. If we assume that the firms play a two-stage game where the cost leading firm first selects the transport technology and then the duopolists simultaneously select prices, two outcomes may arise. If the difference in unit production costs is important, the best transport technology will be chosen and the firms will have markedly unequal market areas. Otherwise the worst transport technology prevails and firms have market areas of similar extent.

If the spatial distribution of demand is very asymmetric, firms locate in equilibrium at the larger market for every value of the transport cost. If the distribution is more balanced two outcomes may arise. With high transport cost each firm's location is oriented towards local demand and firms are spatially dispersed. If the transport cost is low, firms cluster at a central point of customer distribution. The critical level for the transport cost depends inversely on the ratio of the population of the smaller town to the population of the larger one.

Therefore in the case where the population distribution is relatively balanced, the improvement in transport technology may have adverse effects. The decrease of the transport rate with given locations of firms increases quantities consumed by all customers. However when t reaches the critical value where relocation of the firm in the smaller market occurs, the change in transport is not Pareto improving. Although the aggregate consumption does not decrease, the quantity demanded in the smaller market strictly decreases on account of the loss of local production (3).

The two models presented above share the feature that the impact of the improvement of transport on the equilibrium solution is related with the magnitude of asymmetries. While this relation is direct in the case of asymmetry in productive efficiency, with a high level of asymmetry implying a large impact of transportation improvement, it is inverse in the second model. There a strongly asymmetric distribution of the population implies that the decrease of the transport rate, however strong, does not change the locational pattern of firms. The locational impact of the transport improvement occurs only when the spatial distribution of population is relatively balanced.

How do economic asymmetries in the context of decaying transport rates rank from the viewpoint of welfare? The models presented above enable us to say that asymmetries are welfare-enhancing. Production cost asymmetries provide an incentive for the progress of transportation, which otherwise would be nonexistent. On the other hand, strong asymmetry in the spatial distribution of population makes the progress of transport Pareto-improving, while for lower levels of asymmetry this property is absent.

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(3) The result that, with price elastic demand, the firms are dispersed in space in equilibrium when the transport cost is high and tend to cluster at a central point of the spatial distribution of consumers as transport improves, holds for any shape of the distribution (provided it is non-degenerate) and any kind of competition. For instance, Smithies (1941) showed that this result holds if the distribution of consumers is uniform and firms compete with prices.

APPENDIX A

Two-stage game with firm-specific transport rates

t_1 and t_2 are the firm-specific transport rates. Then the market area border x is determined by the equality of delivered prices:

$$p_1 + t_1 x = p_2 + t_2(1 - x) \qquad \textbf{A.1}$$

so that x is:

$$x = \frac{p_2 - p_1 + t_2}{t_1 + t_2} \qquad \textbf{A.2}$$

The profit functions of the firms are:

$$\pi_1(p_1, p_2) = (p_1 - c_1)x = (p_1 - c_1)\frac{(p_2 - p_1 + t_2)}{t_1 + t_2} \qquad \textbf{A.3}$$

$$\pi_2(p_1, p_2) = (p_2 - c_2)x = (p_2 - c_2)\left(1 - \frac{p_2 - p_1 + t_2}{t_1 + t_2}\right) \qquad \textbf{A.4}$$

We assume a two stage game, where the firms in the first stage select simultaneously transport rates and, in the second stage, compete in prices. Transport rates are bounded to the interval $[\,t, t\,]$. Working by backward induction we find a subgame perfect equilibrium.

The first order conditions in the pricing subgame are:

$$\frac{\partial \pi_1}{p_1} = 0$$

$$\frac{\partial \pi_2}{\partial p_2} = 0$$

Together they yield Nash equilibrium prices p^*_1 and p^*_2 as functions of t_1 and t_2:

$$p^*_1(t_1, t_2) = \frac{2t_2 + c_2 + t_1 + 2c_1}{3} \qquad \textbf{A.5}$$

$$p^*_2(t_1, t_2) = \frac{t_2 + 2c_2 + 2t_1 + c_1}{3} \qquad \textbf{A.6}$$

It is clear from **A.5** and **A.6** that p^*_1 and p^*_2 are strictly increasing functions of t_1 and t_2: to increase the transport rates reduces the intensity of price competition.

Substituting **A.5** and **A.6** in **A.2**, we obtain the market area border x at the Nash price equilibrium as a function of the transport rates:

$$x^*(t_1, t_2) = \frac{2t_2 + t_1 + c_2 - c_1}{3(t_1 + t_2)} \qquad \textbf{A.7}$$

It can be easily checked that x^* is a strictly decreasing function of t_1 and a strictly increasing function of t_2. This result is expected: a firm increases its market area by improving its transport technology.

On the other hand, we wish to assess the impact of a general improvement in transport technologies, i. e., we seek to determine the sign of the directional derivative of $x^*(t_1, t_2)$ in the direction $(-1, -1)$. We conclude easily that:

$$\text{sign grad } x^*(t_1, t_2) \cdot (-1, -1) = \text{sign } (t_2 - t_1) + 2(c_2 - c_1) \qquad \textbf{A.8}$$

As $c_2 < c_1$, the directional derivative is negative either if $t_2 < t_1$ or, if $t_2 > t_1$, the module of the diference among the transport rates does not exceed twice the module of the difference among unit production costs. The meaning of this is clear: if the firm with higher production cost does not compensate its productive inefficiency through a marked advantage of efficiency in transportation, an overall improvement in transport shifts consumers from it to the firm with low production cost.

Substituting **A.5**, **A.6** and **A.7** in **A.3** and **A.4**, we obtain the payoff functions in the first stage of the game:

$$\pi_1(t_1, t_2) = \left[p^*_1(t_2, t_1) - c_1\right] x^*(t_1, t_2) = \frac{(2t_2 + c_2 + t_1 - c_1)^2}{9(t_1 + t_2)} \qquad \textbf{A.9}$$

$$\pi_2(t_1, t_2) = \left[p^*_2(t_1, t_2) - c_2\right]\left[1 - \pi^*(t_1, t_2)\right] = \frac{(2t_1 + c_1 + t_2 - c_2)^2}{9(t_1 + t_2)} \qquad \textbf{A.10}$$

We can prove very easily the following lemma.

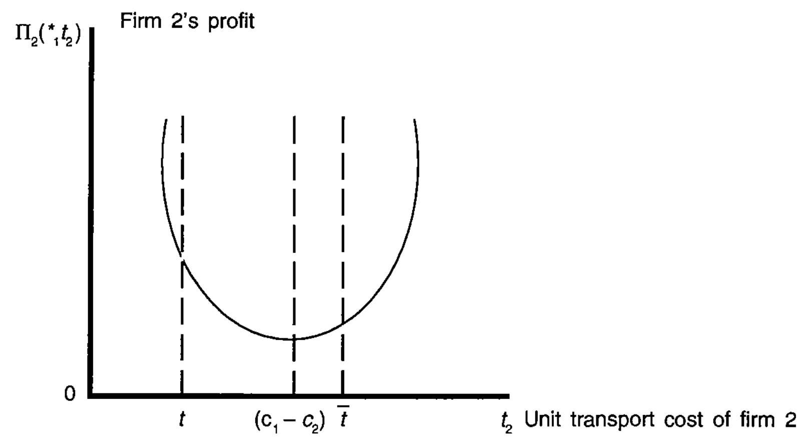
Lemma — In the transport rate subgame the relatively inefficient firm has a dominant strategy which consists in selecting the upper bound transport rate $\bar{t}$.

This lemma follows very easily from the fact that firm 1's profit in **A.9** is strictly increasing in the firm's transport rate. Therefore the solution of the game depends only on firm 2' payoff function in **A.10**. It is easily seen that function $\pi_2(t_1, t_2)$ is convex and has a minimum at:

$$t_2 = c_1 - c_2 \qquad \textbf{A.11}$$

FIGURE A.1

$\pi_2(t_1, t_2)$ function



The maximum of function $\pi_2(t_1, t_2)$ must occur either at t or $\bar{t}$. A sufficient condition so that the more efficient transport technology is selected is:

$$c_1 - c_2 \geq \bar{t} \qquad \textbf{A.12}$$

Whenever it is selected in equilibrium by firm 2, the transport rate t will prevail in most of the market segment, as firm 2 has a market area strictly higher than one half the length of the market. The adoption of an efficient transport technology is therefore dependent on the existence of a sufficient degree of productive asymmetry among the firms [1].

[1] Remark that condition **A.12** is less strict than condition **2.11**.

APPENDIX B

Proof of proposition 1

We assume according to reasons above explained that $t \varepsilon [0,1]$. Firm 1' s total profit function when both firms locate in B is:

$$\pi_1(B,B) = \pi_{1a}(B,B) + \pi_{1b}(B,B) \quad \textbf{B.1}$$

According to **3.8** and **3.4** by symmetry (exchanging subscripts a and b), we have:

$$\pi_{1a}(B,B) = \frac{n_a(1-t)^2}{9} \quad \textbf{B.2}$$

$$\pi_{1b}(B,B) = \frac{n_b}{9} \quad \textbf{B.3}$$

From **B.1**, **B.2** and **B.3**, it follows that firm 1' s total profit when both firms locate in B is:

$$\pi_1(B,B) = \frac{n_a(1-t)^2 + n_b}{9} \quad \textbf{B.4}$$

Assume now that $t \leq \frac{1}{2}$. According to **3.28**, firm 1' s total profit when it locates in A and the rival remains in B is:

$$\pi_1(A,B) = \frac{n_a(1+t)^2 + n_b(1-2t)^2}{9} \quad \textbf{B.5}$$

It is easy to prove that $n_a > n_b$ is a sufficient condition that $\pi_1(A,B) > \pi_1(B,B)$.

Assume now that $t > \frac{1}{2}$. According to **3.23** and **3.25**, firm 1' s profit when firms have different locations is:

$$\pi_1(A,B) = \frac{n_b}{4} \quad \textbf{B.6}$$

We can conclude easily that also in this case $n_a > n_b$ is a sufficient condition that $\pi_1(A,B) = \pi_1(B,B)$. Q. E. D.

APPENDIX C

Proof of proposition 2

For reasons acounted above, we suppose that $t \varepsilon [0,1]$. As a first case, we assume that $t > \frac{1}{2}$. Then firm 2's profit when firms locate in different markets is (according to **3.27** by exchanging subscripts a and *b*):

$$\pi_2(A,B) = \frac{n_b}{4} \tag{C.1}$$

Firm 2' s profit when both firms locate in *A* (according to **3.4** and **3.8**) is:

$$\pi_2(A,A) = \frac{n_a + n_b(1-t)^2}{9} \tag{C.2}$$

Then the condition that $\pi_2(A,B) = \pi_2(A,A)$ is equivalent to

$$t > -\sqrt{\frac{9}{4} - \frac{n_a}{n_b}} \tag{C.3}$$

The r. h. s. of inequality **C.2** is a decreasing and convex function of $\frac{n_b}{n_a}$ over $\left[\frac{4}{9}, \frac{1}{2}\right]$. (A,B) cannot be a location equilibrium if $\frac{n_b}{n_a} < \frac{4}{9}$.

Assume now that $t \leq \frac{1}{2}$. From **B.5** by symmetry (exchanging subscripts a and *b*, and 1 and 2) firm 2' s profit, when it locates in *B* and the rival is based in *A*, becomes:

$$\pi_2(A,B) = \frac{n_b(1+t)^2 + n_a(1-2t)^2}{9} \tag{C.4}$$

On the other hand, from **B.4** by symmetry we get firm 2' s total profit when both firms locate in town *A*:

$$\pi_2(A,A) = \frac{n_b(1-t)^2 + n_a}{9} \tag{C.5}$$

It is easy to show that condition:

$$\pi_2(A,B) \geq \pi_2(A,A)$$

means that:

$$t \geq 1 - \frac{n_b}{n_a} \tag{C.6}$$

Q. E. D.

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